On the Multi-GNSS RTK Positioning Performance in New Zealand

Robert Odolinski (1)
National School of Surveying/University of Otago/New Zealand
+64 3 479 5401 & +64 3 479 7586, robert.odolinski@otago.ac.nz

Paul Denys (2)
National School of Surveying/University of Otago/New Zealand
+64 3 479 7596 & +64 3 479 7586, paul.denys@otago.ac.nz

ABSTRACT

With the advent of new Global Navigation Satellite Systems (GNSSs), multi-system, multi-frequency precise real-time kinematic (RTK) positioning can potentially be possible anywhere, at any time. The satellite constellations include the European Galileo and the Chinese BeiDou Navigation Satellite System (BDS), the regional constellation of Japan’s Quasi-Zenith Satellite System (QZSS) and the modernized American Global Positioning System (GPS). Preliminary positioning results when combining satellites from these CDMA systems have been obtained in Australia. However, the multi-GNSS positioning performance in New Zealand has not yet been investigated. This study aims to give an initial overview of the single-baseline RTK positioning performance achievable in the South Island of New Zealand, a region with a good visibility of all these constellations. Comparisons will be made to the positioning performance obtained in Australia that has a better visibility of the Asia-Pacific regional systems. The between-receiver differential code and phase inter-system biases (ISBs) on the overlapping frequencies will be analysed and used as a-priori corrections on independent baselines, in order to maximize the redundancy of the multi-GNSS RTK functional models. It will be shown that if the ISBs are neglected there can be a serious effect on the ambiguity resolution performance and thus the precision and reliability of the positioning results. It will also be illustrated that by combining all four-systems the RTK positioning results will be significantly improved when compared to using GPS as a stand-alone system, which has the potential to further advance applications which require high precision positioning for GNSS users in New Zealand.

KEYWORDS: Multi-Global Navigation Satellite System (GNSS), real-time kinematic (RTK), integer ambiguity resolution, inter-system biases (ISBs), New Zealand
1. INTRODUCTION

The Global and Regional Navigation Satellite Systems (GNSSs/RNSSs) are expected to be of full-constellations by 2020. The global constellations consist of the American Global Positioning System (GPS), Russian GLONASS, European Galileo and Chinese BeiDou Navigation Satellite System (BDS), whereas the regional constellations are Japan’s Quasi-Zenith Satellite System (QZSS) and the Indian Regional Navigation Satellite System (IRNSS). First results using BDS-only for precise positioning can be found in, e.g., Montenbruck et al. (2013). Real time kinematic (RTK) positioning results when combining these satellite systems have been shown in the literature, e.g., in Deng et al. (2013), He et al. (2013) and Odolinski et al. (2013; 2014a; 2015a) for short to medium baselines, and in Odolinski et al. (2014b; 2015b) for long baselines. We define short baselines to when the baseline lengths are sufficiently small so that the relative ionospheric and tropospheric delays can be neglected, whereas for long baselines the atmospheric delays need to be estimated. All of these studies are based on GNSS data in China or Australia, whereas similar studies in New Zealand have not yet been conducted. This study aims to give a first initial overview of the multi-GNSS RTK performance in Dunedin, New Zealand, and comparisons will be made to the performance that can be obtained in Perth, Australia.

The ground tracks of BDS, Galileo and QZSS (April 29, 2013) as observed from a receiver in Perth Australia with an elevation cut-off angle of 10° are depicted in the left column of Figure 1. The corresponding ground tracks observed by a receiver in Dunedin New Zealand (February 6, 2015) are depicted in the right column of Figure 1. The Perth station is shown to track 5 Geostationary Earth Orbit (GEO), 5 Inclined Geo-Synchronous Orbit (IGSO) and 4 Medium Earth Orbit (MEO) BDS satellites, 3 Galileo In-Orbit Validation (IOV) MEO satellites, and 1 Highly-inclined Elliptical Orbit (HEO) QZSS satellite. The Dunedin station, however, is seen to track fewer satellites over the day (in comparison to Perth), namely 3 GEO and a smaller part of the 5 IGSO/QZSS orbits, and 3 Galileo MEO satellites. Thus the single-baseline RTK positioning performance improvement is expected to be less significant in Dunedin as compared to Perth when combining these satellite systems with GPS.

During the time of writing, four additional Galileo Full Operational Capability (FOC) MEO satellites have been launched, with their corresponding signals expected to be available for civilian users by the middle of 2015 (see e.g. GPS World, 2015a). A sixth IGSO BDS satellite has also been launched as of March 30, 2015 (see e.g. GPS World 2015b), being the first BDS satellite with a B1 frequency centred at the L1/L1 GPS/QZSS and E1 Galileo frequency. At the time of writing, however, this BDS satellite was also not available for positioning. The
IRNSS satellites are not visible from New Zealand, but some first positioning results using this system can be found in e.g. Nadarajah et al. (2015). The Russian GLONASS is not included herein since it only has one satellite available that is based on the Code Division Multiple Access (CDMA), which tracks a frequency that does not overlap any of the other frequencies used in this study.

In this study we will focus on the single-frequency instantaneous RTK model for short baselines. The advantage with single-frequency RTK is that less expensive receivers can potentially be used, and instantaneous RTK allows for a model that becomes insensitive to cycle-slips. The frequencies that will be analysed are L1 from GPS, E1 from Galileo, L1 from QZSS and B1 from BDS so as to maximize the number of satellites with overlapping frequencies. The E1 Galileo and L1 GPS/QZSS frequencies are overlapping. However the future BDS satellites will contain a B1 frequency that will overlap the other frequencies as well (GPS World 2015b), thus allowing for an even stronger functional model when combining all four systems (see Table 1).

In Section 2 we describe the full-rank between-receiver single-differenced RTK model for combining BDS, Galileo, QZSS and GPS. We distinguish between the inter-system bias (ISB) float model, where the ISBs are estimated, and the ISBs-fixed model, where the ISBs are a priori corrected in order to strengthen the model. In Section 3 the multi-GNSS RTK positioning performance is analysed and compared between Perth, Australia and Dunedin, New Zealand. In Section 4 we end our study with a summary and conclusions.

2. MULTI-GNSS SINGLE-BASELINE RTK FUNCTIONAL MODELS

The four-system functional RTK model employed in this study was initially derived and presented in Odolinski et al. (2015a), based on between-receiver single-differenced (SD) observations. This model was derived through S-system theory, which implies null-space identification, S-basis constraining and further interpretation of the estimable unknowns, see e.g. Teunissen (1985). The SDs eliminate satellite-dependent parameters, such as satellite clocks, satellite hardware code/phase and initial phase delays, and provided that the receiver clock is shared among the systems the time offsets are also eliminated. Since we assume short baselines of less than a few kilometres, any remaining satellite orbit errors and relative atmospheric delays can be neglected. For a four-system RTK model that is suitable for long baselines where the relative atmospheric delays need to be estimated, see e.g. Odolinski et al. (2014b).

2.1 Four-system ISBs-float functional RTK model

The “ISBs-float” model implies that the inter-system biases (ISBs) will be estimated. We consider \( r = 1, 2 \) receivers tracking the GPS satellites \( s_G = 1_G, \ldots, m_G \) and GNSS system * satellites \( s_* = 1_*, \ldots, m_* \), where * represents either (E) Galileo, (Q) QZSS or (B) BDS. For notational convenience we refrain from carrying through SD random observation noise and un-modelled effects such as multipath. The linearized full-rank SD system of observation equations assuming overlapping frequencies \( j = 1, \ldots, f \) then reads (in units of range),

\[
\begin{align*}
\mathbf{p}^G_{12,j} &= -\mathbf{c}_2^G \Delta \mathbf{x}_{12} + \mathbf{d}_{12,j}^G + \mathbf{a}_{12,j}^G \\
\mathbf{\phi}^G_{12,j} &= -\mathbf{c}_2^G \Delta \mathbf{x}_{12} + \mathbf{d}_{12,j}^G + \mathbf{\lambda}_{j}^G \mathbf{z}_{12,j}^G \\
\mathbf{p}^*_j &= -\mathbf{c}_2^* \Delta \mathbf{x}_{12} + \mathbf{d}_{12,j}^* + \mathbf{a}_{12,j}^* \\
\mathbf{\phi}^*_j &= -\mathbf{c}_2^* \Delta \mathbf{x}_{12} + \mathbf{d}_{12,j}^* + \mathbf{\lambda}_{j}^* \mathbf{z}_{12,j}^*
\end{align*}
\]  

(1)
where \((\cdot \big| \cdot)\) is the between-receiver SDs notation, \(p_{l2,j}^s\), \(\phi_{l2,j}^s\) are the SD code/phase observables respectively, \(c_{\nu}^{\|} = (x^\nu - x_r)^T / \|x^\nu - x_r\|\) is the line-of-sight unit vector from the receiver \(r\) to the GNSS satellites \(s\) obtained from linearizing the system of equations with respect to the receiver coordinates, \((\cdot)^T\) is the transpose of a vector, \(\|\cdot\|\) denotes the norm, \(x^\nu\) is the vector of satellite coordinates and \(x_r\) the vector of receiver coordinates, and \(\lambda_j\) is the wavelength corresponding to frequency \(j\). The estimable unknowns are,

\[
\begin{align*}
\Delta x_{12} &= \Delta x_2 - \Delta x_1 & \text{relative receiver coordinates}, \\
d_{l2} &= d_{l2} + d_{l2,j}^G & \text{relative receiver clock with GPS differential code delay}, \\
\tilde{d}_{l2,j}^G &= d_{l2,j}^G - \tilde{d}_{l2,j}^G & \text{relative GPS Differential Code Bias (DCB) estimable for \(j > l\)}, \\
\delta_{l2,j}^G &= \delta_{l2,j}^G - d_{l2,j}^G + \lambda_j z_{l2,j}^G & \text{relative GPS receiver hardware (HW) phase delay}, \\
\tilde{d}_{l2,j}^o &= \tilde{d}_{l2,j}^o - \tilde{d}_{l2,j}^o & \text{differential code ISB}, \\
\tilde{\delta}_{l2,j}^o &= \tilde{\delta}_{l2,j}^o - \tilde{\delta}_{l2,j}^o + \lambda_j z_{l2,j}^G & \text{differential phase ISB biased by a double-differenced (DD) inter-system ambiguity}, \\
z_{l2,j}^{c,s} &= z_{l2,j}^{c,s} - z_{l2,j}^{c,s} & \text{DD GPS integer ambiguity}, \\
z_{l2,j}^{l,c} &= z_{l2,j}^{l,c} - z_{l2,j}^{l,c} & \text{DD BDS, Galileo or QZSS integer ambiguity}.
\end{align*}
\]

The ambiguities are integers since they are implicitly double-differenced and thus the initial receiver and satellite HW phase delays have been eliminated. The number of observations, estimable unknowns and redundancy of the model in (1) is shown in Table 1.

### 2.2 Four-system ISBs-fixed functional RTK model

The “ISBs-fixed” model implies that the inter-system biases (ISBs) are completely known and thus can be subtracted as a priori corrections in the code and phase observations in (1) to strengthen the model accordingly. It can be shown (see Odolinski et al. 2015a) that the full-rank SD system of observation equations are then obtained as follows,

\[
\begin{align*}
p_{l2,j}^G &= -c_{\nu}^{\|} \Delta x_{12} + d_{l2} + \tilde{d}_{l2,j}^G \\
\phi_{l2,j}^G &= -c_{\nu}^{\|} \Delta x_{12} + d_{l2} + \tilde{\delta}_{l2,j}^G + \lambda_j z_{l2,j}^{c,s} \\
p_{l2,j}^* &= d_{l2,j}^o - \tilde{d}_{l2,j}^o - \Delta x_{12} + d_{l2} + \tilde{d}_{l2,j}^G \\
\phi_{l2,j}^* &= d_{l2,j}^o - \tilde{\delta}_{l2,j}^o - \Delta x_{12} + d_{l2} + \tilde{\delta}_{l2,j}^G + \lambda_j z_{l2,j}^{c,s}
\end{align*}
\]

(2)

where \(\tilde{d}_{l2,j}^o\) denotes the code ISBs corrections and \(\tilde{\delta}_{l2,j}^o\) the corresponding phase ISBs corrections. In Equation (2) we have the following estimable integer ambiguities for system *,

\[
\tilde{z}_{l2,j}^{c,s} = z_{l2,j}^{c,s} - z_{l2,j}^{c,s} - a_{l2,j}
\]

(3)

The ambiguity is now differenced with respect to the pivot GPS satellite \(I_G\) ambiguity minus an integer ambiguity \(a_{l2,j}\) that stems from the observations to determine the phase ISB corrections. The redundancy corresponding to Equation (2) is shown in Table 1.

### 2.3 Redundancy and solvability condition of the four-system functional RTK models

The number of observations, estimable unknowns and redundancy for the full-rank four-system single-baseline RTK models in (1) and (2) is presented in Table 1. Last columns depicts the solvability condition, which is the number of satellites required to solve the
positioning models. It will further be used to evaluate the positioning availability in the following results section. For example, Table 1 implies that when we have a single-system or a four-system ISBs-fixed model at least four satellites are needed for positioning provided that all frequencies overlap. Note when the ISBs-float four-system model is used, at least seven satellites are needed. This shows the importance of having ISBs corrections available since it will increase the positioning availability in urban canyon environments where higher satellite elevation cut-off angles are needed, and strengthens the model because of the increase in redundancy. However since B1 of BDS does currently not overlap L1 of GPS, we need five satellites for the four-system model that is evaluated in the coming section (shown in the last row of Table 1).

Table 1 Single-epoch, single-baseline RTK redundancy and solvability condition. The last row corresponds to the ISBs-fixed four-system RTK model used in the following sections (Table from Odolinski et al 2015a). Note that \( m \) is the number of satellites and \( f \) the number of frequencies for GPS (G), BDS (B), Galileo (E) and QZSS (Q) respectively

<table>
<thead>
<tr>
<th>Model</th>
<th># of observations</th>
<th># of unknowns</th>
<th>Redundancy</th>
<th>Solvability condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-system</td>
<td>2f+2m+</td>
<td>3f+3m+</td>
<td>( f(m-1)-3 )</td>
<td>( m\geq4 )</td>
</tr>
<tr>
<td>4-system ISBs-float (1)</td>
<td>2fG+2fB+</td>
<td>3fG+3fB+</td>
<td>( f(G-1)+f(B-1) )</td>
<td>( G+mG+mB )</td>
</tr>
<tr>
<td></td>
<td>+2fE+2fQ</td>
<td>+fE+fQ</td>
<td>( +f(G-1)+f(E-1) )</td>
<td>( G+E+mE+mQ )</td>
</tr>
<tr>
<td>4-system, ISBs-fixed assuming all frequencies overlap (2)</td>
<td>2fG+2fB+</td>
<td>3fG+fB+</td>
<td>( f(G-1) )</td>
<td>( G+mG+mB )</td>
</tr>
<tr>
<td></td>
<td>+2fE+2fQ</td>
<td>+E+fQ</td>
<td>( +f(G-1)+f(E-1) )</td>
<td>( E+mE+mQ )</td>
</tr>
<tr>
<td>4-system, ISBs-fixed for the overlapping frequencies used herein</td>
<td>2fG+2fB+</td>
<td>3fG+fB+</td>
<td>( f(G-1)+f(B-1) )</td>
<td>( G+mG+mB )</td>
</tr>
<tr>
<td></td>
<td>+2fE+2fQ</td>
<td>+E+fQ</td>
<td>( +f(G-1)+f(E-1) )</td>
<td>( E+mE+mQ )</td>
</tr>
</tbody>
</table>

3. MIXED-RECEIVER MULTI-GNSS RTK IN NEW ZEALAND AND AUSTRALIA

This section evaluates the performance of the four-system RTK models in (1) and (2) for two days of real data in Perth (2013) and Dunedin (2015). Comparisons will be made between the two locations as well as to the performance when one uses GPS separately or some other combinations of the systems. Emphasis will be on different types of receivers, e.g. Trimble NetR9 and Septentrio Polarx4. We will distinguish between a formal and empirical analysis. The formal analysis is based on the design matrix and variance-covariance (VCV) matrix of the observations only, i.e. real data is not necessary, whereas the empirical analysis involves real data to show the actual performance of the four-system RTK models.

3.1 Data collection

The zero baseline that will be used for the inter-system bias (ISB) calibration is depicted in Figure 2, which makes use of Equation (1) but with the receiver positions fixed. The zero baseline consists of two receivers (Trimble NetR9, CUT0, and Septentrio Polarx4, CUT1) connected to the same antenna, as to eliminate any multipath effects. If the ISBs estimated from this baseline are time constant they can be used as a priori corrections on independent single-baselines with data collected one month later in Perth (CUT1-CUTT in Figure 2) and two years later in Dunedin (see Figure 3), in order to maximize the redundancy of the RTK models. The baseline distances that will be evaluated are less than one kilometre, which allows us to neglect the relative atmospheric delays. The stochastic model settings (Table 2) are based on an exponential elevation weighting function by Euler and Goad (1991) and the zenith-referenced a priori code and phase standard deviations (STDs) for undifferenced observations. The STDs used in Perth are depicted within brackets. The stochastic model was based on data independent of the data used in this study.
Table 2: Zenith-referenced code/phase a priori standard deviations for Trimble NetR9 to Septentrio Polarx4 when evaluating the multi-GNSS RTK performance in Dunedin (Perth within brackets)

<table>
<thead>
<tr>
<th>Sat. system</th>
<th>Frequency</th>
<th>Code [cm]</th>
<th>Phase [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS</td>
<td>L1</td>
<td>25 (30)</td>
<td>2 (2)</td>
</tr>
<tr>
<td>BDS</td>
<td>B1</td>
<td>30 (35)</td>
<td>2 (2)</td>
</tr>
<tr>
<td>Galileo</td>
<td>E1</td>
<td>30 (30)</td>
<td>2 (2)</td>
</tr>
<tr>
<td>QZSS</td>
<td>L1</td>
<td>30 (30)</td>
<td>3 (3)</td>
</tr>
</tbody>
</table>

The total number of satellites tracked by the receivers in Perth over one day of data (2013) is depicted in Figure 4 (a) for an elevation cut-off angle of 10°, and the corresponding satellite visibility in Dunedin (2015) is given in Figure 4 (b). Although one less Galileo IOV satellite was tracked by the receivers in Dunedin, we can still see nearly double the number of satellites for a combined four-system model (black line) in Dunedin in comparison to using only GPS (blue line) over the entire day. However, because of the better visibility of the Asia-Pacific regional constellations in Perth, the total number of satellites is larger when compared to Dunedin for most of the day (see also Figure 1).
Satellite visibility in Perth

Satellite visibility in Dunedin

Figure 4: Satellite visibility: (a) Perth (April 20, 2013), (b) Dunedin (February 6, 2015) for an elevation cut-off angle of 10°

The LAMBDA method is used for ambiguity resolution (Teunissen 1995), and the detection, identification and adaptation (DIA) procedure is employed to eliminate any outliers (Teunissen 1990). Since use is made of Trimble-Septentrio receivers, the BDS phase inter-satellite-type biases (ISTBs) of 0.5 cycles between GEO and IGSO/MEO satellites (see Nadarajah et al., 2014) have been applied to the phase observations of the GEO satellites, since otherwise the integer ambiguity resolution performance was shown to become catastrophically poor.

The daily average of the ISBs estimated in Perth by the zero-baseline setup (Figure 2) were shown to be consistent within two days separated by one year (2013 vs 2014). It is also demonstrated in the following sections that the ISBs corrections applied to the receiver-pairs in Perth (2013) are applicable for the independent receiver-pairs in Dunedin (2015), see e.g. Table 3. However a recent study (Zhang and Teunissen, 2015) shows that the multi-GNSS between-receiver differential code biases (DCBs) daily weighted averages can vary over a 1-year interval. This has been attributed to firmware upgrades and daily maximum temperature variations at the receiver sites. Changes in firmware and temperature might thus also affect the code ISBs due to their similar mathematical definition, see Equation (1). Therefore it is stressed that proper monitoring of the ISBs are required over long time spans and varying conditions.

3.2 Formal analysis of mixed-receiver multi-GNSS single-baseline RTK performance

Based on the VCV-matrix of the (decorrelated) float ambiguities we compute the bootstrapped success rate (SR), which is a precise lower bound to the integer least squares (ILS) SR (Teunissen 1998). Since it is a formal measure, it can be used before any actual GNSS measurements are collected and to predict whether ambiguity resolution is expected to be successful.

The single-epoch bootstrapped SRs for elevation cut-off angles ranging between 10-35° are depicted in Figure 5 based on the four satellite constellations over two days of data in Perth (April 29-30, 2013) at top, and Dunedin (February 6-7, 2015) at bottom. L1 GPS-only RTK is depicted in blue, E1+L1 Galileo+GPS in green, E1+L1+L1 Galileo+QZSS+GPS in red, B1+L1 BDS+GPS in cyan and a four-system B1+E1+L1+L1 RTK model in black. Dotted
lines are ISBs-float models and the ISBs-fixed models are depicted as full lines when applicable. Note that the ISBs-float models represented by red and black dotted lines are a dual and three system model respectively, since the single QZSS satellite can then not contribute to the model (as it is required to estimate the ISBs). Since the number of GPS satellites is overall smaller and one less Galileo satellite could be tracked in Dunedin (2015) when compared to Perth (2013), see Figure 4, we depict in Figure 5 the SRs for a E1+L1+L1 Galileo+QZSS+GPS RTK model (red lines) in Dunedin at bottom. This is done as to make a fairer comparison to the E1+L1 Galileo+GPS RTK model (green lines) in Perth at top of Figure 5.

![Figure 5: Formal single-epoch bootstrapped SR as a function of elevation cut-off angles between 10-35°. The bootstrapped SRs are taken as a mean over April 29-30, 2013 for Perth (a), and February 6-7, 2015 for Dunedin (b).](image)

The L1 GPS-only SRs in Figure 5 are overall larger for all cut-off angles in Dunedin as compared to Perth which is related to the better GPS code precision for the two receivers in Dunedin (see Table 2). Note also that the E1+L1 Galileo+GPS ISBs-float model SRs in Perth (dotted green line) resembles the E1+L1+L1 Galileo+QZSS+GPS ISBs-float model SRs in Dunedin (dotted red line) for all cut-off angles. More importantly the SRs increase for both these models when the ISBs are fixed (green and red full lines), particularly for Dunedin. The additional Galileo and single QZSS satellites contribute significantly to the solution when the total number of satellites are small. This can also be seen by inspecting the four-system models (dotted and full black lines), where the SRs increase more significantly for higher cut-off angles when the ISBs are fixed. We can thus conclude that ISB-calibration is particularly important in environments with restricted satellite visibility.

The SRs in Figure 5 become spectacularly large when the BDS constellation is added to GPS, where the SRs are close to 100% in Perth for elevation cut-off angles up to 25° and the BDS+GPS and four-system models. The corresponding cut-off angles in Dunedin is 10-20° for the four-system model and provided that the ISBs are fixed (full black line). The difference in performance with respect to Perth is mainly due to the fewer number of BDS satellites tracked in Dunedin (see Figure 1 and Figure 4). Compare also the four-system SRs
of 100% to the single-system GPS-only SRs, which, e.g., for the cut-off angle of 20° are close to 50% for both locations.

### 3.3 Empirical analysis of mixed-receiver multi-GNSS single-baseline RTK performance

In this section real data is used to verify the formal claims in the previous section and show the actual performance of the four-system RTK models. The empirical integer least squares (ILS) success rate (SR) is computed by comparing the single-epoch estimated integer ambiguities to a set of reference ambiguities. These reference ambiguity were computed by using a four-system multi-frequency RTK model with fixed receiver positions, a Kalman filter and a dynamic model, where the ambiguities are treated as time constant over the entire observation time span of two days. The number of correctly fixed epochs divided by the total number of epochs of positioning solutions are then the ILS SR. The single-epoch ILS SRs are depicted in Table 3 for the different combination of systems as depicted in Figure 5 for both locations, where SRs of 100% are given in bold.

The ISBs-fixed model is depicted with ‘a’ in the second column of Table 3, and the ISBs-ignored model is depicted by a ‘b’. The ISBs-ignored model is equivalent to the ISBs-fixed model in Equation (2) except that the ISBs and code/phase ISBs corrections are then incorrectly assumed to be zero for the mixed receiver types. There is a dramatic decrease in SRs when the ISBs are ignored (‘b’ (red) and Table 3) compared to when they are corrected (‘a’). Two examples of the corresponding empirical RTK positioning results when ignoring and correcting for the ISBs will be shown in the following section. The ISBs-float model (1) is finally given as ‘c’ and is shown to provide for significantly larger SRs in comparison to when the ISBs are ignored, but somewhat smaller than for the ISBs-fixed model due to its smaller redundancy (see Table 1).

<table>
<thead>
<tr>
<th>Location and system/frequency</th>
<th>ISBs</th>
<th>Empirical ILS SR [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>cutoff [°]:</td>
<td>10</td>
</tr>
<tr>
<td>Perth</td>
<td>GPS L1</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>E1+L1 Galileo+GPS</td>
<td>a</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c</td>
</tr>
<tr>
<td></td>
<td>B1+L1 BDS+GPS</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>B1+E1+L1+L1</td>
<td>a</td>
</tr>
<tr>
<td></td>
<td>BDS+Galileo+QZSS+GPS</td>
<td>b</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c</td>
</tr>
<tr>
<td>Dunedin</td>
<td>GPS L1</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>E1+L1+L1 Galileo+QZSS+GPS</td>
<td>a</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b</td>
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<tr>
<td></td>
<td></td>
<td>c</td>
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<tr>
<td></td>
<td>B1+L1 BDS+GPS</td>
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<td></td>
<td>B1+E1+L1+L1</td>
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<td></td>
<td>BDS+Galileo+QZSS+GPS</td>
<td>b</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c</td>
</tr>
</tbody>
</table>

More importantly Table 3, which is based on real data, verifies the formal claims made in Figure 5, since almost all ILS SRs are larger than the corresponding bootstrapped SRs.
However this does not hold true for the elevation cut-off angle of 10°, and when investigating these incorrectly fixed instances for both Perth and Dunedin we found that they were related to low-elevation multipath for a few GPS satellites that were rising/setting. This in addition to the GEO C03 BDS satellite (see Figure 1) that caused the majority of the incorrectly fixed instances in Dunedin due to it being almost stationary and having a low elevation angle of around 12° with respect to the receivers. This was also verified by excluding the C03 satellite in Dunedin, and the four-system ISBs-fixed model could then achieve an ILS SR of 99.9% (compare to a ILS SR in Table 3 of 97.9% when including it). However Table 3 illustrates that by combining the satellite systems higher elevation cut-off angles than 10° are allowed where one can still achieve continuous and successful ambiguity resolution. For example for the Perth receivers we are allowed to increase the cut-off angle to 25° and can still obtain 100% ILS SR for the B1+L1 BDS+GPS and four-system models, respectively. The corresponding cut-off angle in Dunedin is 20°, where only the ISBs-fixed four-system model can achieve an ILS SR of 100% as was also predicted by the bootstrapped SR in Figure 5.

3.4 Positioning when ISBs are ignored and fixed respectively

In this section we will show the serious effect ignored ISBs can have on RTK positioning performance when using mixed-receiver types (Trimble NetR9 and Septentrio Polarx4). In Section 3.3 we showed empirically that when the ISBs were ignored, the empirical ILS SR (Table 3) decreased significantly as compared to when they are estimated (ISBs-float model) or corrected (ISBs-fixed model).

Figure 6 shows RTK positioning results for L1 GPS (left column), E1+L1 Galileo+GPS ISB-ignored (middle column) and ISBs-fixed model (right column), respectively, for an elevation cut-off angle of 10° in Perth over two days of data (2013). The a-priori ISB corrections were determined from the independent zero-baseline data set one month earlier and subtracted as constant values from the code and phase observation equations respectively. The correctly fixed position solutions are given in green, incorrectly fixed in red, and float solutions in grey colour for the horizontal and vertical positioning scatter (top two rows). We also show the total number of satellites in light green colour (and in red colour when below 8) as to show the relation between the correctly/incorrectly fixed solutions and the number of satellites. In dark green the number of Galileo satellites are depicted to demonstrate the redundancy difference to the GPS-only model, as well as the relation between incorrectly fixed solutions and when the ISBs are neglected.

Figure 6 illustrates that the correctly fixed solutions are two-orders of magnitude more precise at the millimetre-centimetre level when compared to the incorrectly fixed and float position solutions at the decimetre-metre level. We also see in Figure 6 that nearly all epochs are incorrectly fixed when the ISBs are neglected for the E1+L1 Galileo+GPS model (and thus when we have Galileo satellites available). The corresponding ILS SR is 37.4%, see Table 3. However combining Galileo with GPS provides for improved ILS SRs when compared to GPS-only after the ISBs are a priori corrected (92.7% vs 84.5% ILS SR), because of the increase in the number of satellites. In Figure 7, the corresponding L1 GPS and E1+L1+L1 Galileo+QZSS+GPS RTK positioning results are given for Dunedin using two days of data (2015).
Figure 6: L1 GPS (left column) with 84.5% ILS SR, E1+L1 Galileo+GPS ISBs-ignored (middle column) with 37.4% ILS SR, and E1+L1 Galileo+GPS ISBs-fixed (right column) with 92.7% ILS SR. The single-epoch RTK positioning results are for Perth (April 29-30, 2013). The float (gray), incorrectly fixed (red) and correctly fixed (green) solutions are given in local North, East and Up errors and for a cut-off angle of 10°. Total # of satellites is given in light green (in red when below eight and # of Galileo satellites is given in dark green).

Figure 7: L1 GPS (left column) with 90.1% ILS SR, E1+L1+L1 Galileo+QZSS+GPS ISBs-ignored (middle column) with 62.1% ILS SR, and E1+L1+L1 Galileo+QZSS+GPS ISBs-fixed (right column) with 96.6% ILS SR. The single-epoch RTK positioning results are for Dunedin (February 6-7, 2015). The float (gray), incorrectly fixed (red) and correctly fixed (green) solutions are given in local North, East and Up errors and for a cut-off angle of 10°. The # of Galileo/QZSS satellites is depicted in dark green and cyan respectively.

Figure 7 shows a similar catastrophic behaviour as in Figure 6 for the ILS SRs and thus corresponding positioning results when the ISBs are ignored. The ILS SR of 90.1% for GPS-only namely decreases to 62.1% when combining Galileo+QZSS+GPS. Most importantly the ILS SR increases to 96.6% for the same model when all ISBs are correctly fixed.

3.5 Positioning for higher elevation cut-off angles
In the previous sections we concluded that for Perth we were allowed to increase the elevation cut-off angle up to 25° (almost 30°) and could still achieve continuous successful RTK positioning over two days. This was provided that BDS+GPS or all four-systems were combined. The corresponding cut-off angle in Dunedin was 20° (see Table 3). It is thus of
interest to see what happens to the RTK performance when the elevation cut-off angles are increased even further. In Figure 8 we depict the instantaneous RTK positioning results for an elevation cut-off angle of 25° in Dunedin over two days of data (2015). The models shown are L1 GPS (left column), B1+L1 BDS+GPS (middle column) and the four-system ISBs-fixed model (right column). At bottom of the positioning results we depict Positional Dilution of Precisions (PDOPs) for GPS and BDS+GPS to show the relation between large excursions in the positioning errors and when the receiver-satellite geometry is poor. The four-system does not suffer from large PDOPs and thus we plot the number of satellites only instead to show its relation to the amount of incorrectly/correctly fixed solutions.

Figure 8 shows several drawbacks of using a single-system for RTK positioning with higher than customary elevation cut-off angles. The number of satellites is namely not always sufficient to solve the positions over the two days for L1 GPS-only (97.3% positioning availability which was computed from the solvability condition in Table 1). The ILS SR for GPS-only is moreover only 40.5% and some of the positioning solutions suffer from the poor receiver-satellite geometry as illustrated by the very large PDOP values. Adding BDS increases the positioning availability to 100% in both cases, the ILS SRs for BDS+GPS is 88.9% and the four-system model achieves 97.2%. When comparing the ILS SRs for Dunedin to the ones obtained in Perth (Table 3), the four-system models are shown to have comparable ILS SRs for an elevation cut-off angle up to 25° in Dunedin and 35° in Perth.

4. CONCLUSIONS
In this study we have analysed the single-frequency performance of instantaneous RTK positioning when combining satellites from BDS, Galileo, QZSS and GPS for two days of data in Dunedin, New Zealand (2015) and Perth, Australia (2013). In both locations the baseline lengths were below one kilometre so that the relative atmospheric delays can be neglected. Future studies on the New Zealand RTK performance will involve longer baselines where the atmospheric delays need to be estimated, similar to the studies in Odolinski et al. (2014b; 2015b). The conclusions can be summarized as follows.
The overlapping frequencies of E1 Galileo, L1 GPS and L1 QZSS were analysed, in addition to the B1 frequency of BDS. Mixed-receiver types (Trimble NetR9-Septentrio Polarx4) were then used to estimate differential code/phase inter-system biases (ISBs) for the purpose of ISB-calibration on baselines based on independent data one month (Perth, 2013) and two years (Dunedin, 2015) later, respectively. This allows for a functional model where each additional satellite to GPS will then contribute to the solution (the ISBs-fixed model).

A formal and an empirical analysis showed that the four-system ISBs-fixed model achieved larger success rates and thus better precise positioning availability when compared to the single-, dual- or triple-systems. We also concluded that the empirical integer least squares (ILS) success rates when the ISBs were neglected resulted in a poor RTK positioning performance. Whereas when we made use of a priori ISB corrections the ILS success rates were significantly increased in comparison to the ISBs-ignored/ISBs-float counterparts, and the single-system.

We concluded that some of the GPS-only RTK positioning solutions were spoiled by the poor receiver-satellite geometry (elevation cut-off angle of 25°), whereas when BDS+Galileo+QZSS was added the geometry and thus positioning performance was significantly improved. We also concluded that the four-system ISBs-fixed RTK performance in Perth and for an elevation cut-off angle of 35° is comparable to the corresponding performance in Dunedin for a cut-off angle of 25°, and the difference in performance for higher cut-off angles between the two locations can be attributed to the fewer number of BDS satellites tracked in Dunedin. Most importantly we have shown (for the first time) that the ILS success rate for Dunedin is increased from 57.7% for GPS-only to 100% for the four-system ISBs-fixed model and a cut-off angle of 20°. Combining all systems thus has the potential to further advance precise positioning applications for New Zealand GNSS users.

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