International Global Navigation Satellite Systems Society
IGNSS Symposium 2015

Outrigger Gold Coast, Qld Australia
14-16 July, 2015

Network Joint Clock Synchronization and Ranging: Linear Bayesian Solution

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ABSTRACT

Time synchronization and ranging are two critical issues for time of arrival (TOA) based localization systems such as the wireless sensor networks (WSNs), indoor positioning systems and pseudolite systems. In order to jointly estimate the clock skews, clock offsets and fixed relative distances, a two-way timing stamp exchange mechanism to collect the necessary time information is first discussed. Second, a network wide observation formula is derived, according to which, the implementation conditions and restrictions of the existing algorithms apply. Finally, a novel global linear Bayesian (GLB) estimation algorithm is presented in this paper. Unlike the other existing solutions, the proposed algorithm could compute the desired unknown parameters in a recursive, rapid and low-complexity way, and also free the restrictions of symmetric and synchronous communications. The Monte Carlo experiments shows that the GLB estimator achieves the best performance compared with the other existing estimators.

KEYWORDS: Time synchronization, two-way communication, clock model, Bayesian estimation, joint estimation

1. INTRODUCTION

Recently, a number of joint ranging and time synchronization algorithms have been proposed, most of which employ two-way communication to realize the TOA message exchange. Some widely used technologies such as the timing-sync protocol for sensor networks (TPSN) (General et al., 2003) correct the clock offsets but cannot estimate the clock skews. There is no doubt that the synchronization precision would be improved if both the clock offsets and clock skews could be estimated. In addition, the delays between the members are usually unknown for a larger scale system or a low cost system. Noh et al (2007) proposed a maximum likelihood estimator (MLE) based algorithm to estimate the pairwise clock offset and clock skew, where a differencing operation to eliminate the unknown delay is employed. Leng and Wu (2010) proposed a low-complexity least square (LCLS) solution by adding the pairwise two-way observations. It is noted that these two estimators cannot compute the ranging results. The reason is that the ranging term is eliminated for the purpose of saving memory. Rajan and Van der Veen (2011) proposed a global least square (GLS) solution to jointly estimate the clock offset, clock skew and relative distance, which is an extension of the method proposed by Leng. Furthermore, GLS could achieve a higher estimation precision compared with the MLE based methods, due to encompassing the communication information of each node. When the ranging and synchronization result is calculated, localization could also be achieved by the subsequent processing (Chepuri et al., 2012).

It is noted that the solutions mentioned above require a certain time to collect the information of a set number of communication links, namely the idle time of the estimator would be much longer than the processing time. On the other hand, the estimation precision is proportional to the number of links, which implies that the more precise estimation is achieved, the larger size of the information matrices is required. However, the size of the matrices cannot be increased arbitrarily in practice, owing to the computation rate and the memory limitation of hardware.

In this paper, a novel centralized global linear Bayesian (GLB) estimator is proposed to jointly estimate the clock offset, clock skew and fixed distance. The estimator is capable of updating the estimations once any communication link in the network occurs, and it does not put any constraint on the order of links.

Notation: The Hadamard product (division) is denoted by $\odot$ ($\oslash$); the Kroneker product is denoted by $\otimes$; the transposition is denoted by $(\cdot)^T$; the matrices are denoted by boldface letters; The $N \times 1$ vector of ones (zeros) is indicated by $\mathbf{1}_N$ ($\mathbf{0}_N$), $\mathbf{v}_{2N} = [-1,1,-1 \ldots 1]^T \in \mathbb{R}^{2N\times 1}$, and $\mathbf{I}_N$ denotes an identity matrix of size N.

2. SYSTEM MODEL

Consider a distributed network consisting of N nodes, each of which employs a clock as its frequency source. All the network members suffer from clock skews and clock offsets due to the clock uncertainty. Let t be the global time and t_i be the local time of node i, then the relation between t and t_i could be expressed by

$$t_i = w_i t + \varphi_i \tag{1}$$

$$C_i\left(t_i\right) \triangleq t = \alpha_i t_i + \beta_i \tag{2}$$

where w_i, φ_i denote the clock skew and the clock offset of node i respectively, $C_i(t_i)$ denotes the global time of node t at local timet_i. Rearranging (2) and comparing with (1) yields

$$\mathbf{w} = \mathbf{1}_N \oslash \boldsymbol{\alpha} \tag{3}$$

$$\boldsymbol{\varphi} = -\boldsymbol{\beta} \oslash \boldsymbol{\alpha} \tag{4}$$

where $\mathbf{w} = \{w_1, w_2, ..., w_N\}^T$, $\boldsymbol{\varphi} = \{\varphi_1, \varphi_2, ..., \varphi_N\}^T$, $\boldsymbol{\beta} = \{\beta_1, \beta_2, ..., \beta_N\}^T$ and $\boldsymbol{\alpha} = \{\alpha_1, \alpha_2, ..., \alpha_N\}^T$.

The two-way timing message exchange mechanism with a fully asynchronous scenario, as shown in Figure 1, is implemented to realize time synchronization and ranging (Rajan & Van der Veen, 2012). For the *k*th pair of forward link and reverse link between node i and node j, four timing stamps $\{T_{1,ij}^k, T_{2,ij}^k, T_{3,ij}^k, T_{4,ij}^k\}$ can be collected in turn. Each forward link and reverse link can be modelled as

$$C_j(T_{2,ij}^k + n_1^k) - C_i(T_{1,ij}^k + n_2^k) = \tau_{i,j} + n_3^k \tag{5}$$

$$C_i(T_{4,ij}^k + n_4^k) - C_j(T_{3,ij}^k + n_5^k) = \tau_{i,j} + n_6^k \tag{6}$$

where $\{n_1, n_2, n_3, n_4, n_5, n_6\} \sim N(0, \sigma^2)$ are aggregate Gaussian variables originate from measurements and space disturbances and $\tau_{i,j}$ denotes the relative fixed propagation delay between i and j, $\tau_{i,j} = \tau_{j,i}$. Substituting (2) into (5) and (6) yields

$$\alpha_i T_{1,ij}^k = \alpha_j T_{2,ij}^k - \beta_i + \beta_j - \tau_{i,j} - \alpha_i n_1^k + \alpha_j n_2^k - n_3^k \tag{7}$$

$$\alpha_i T_{4,ij}^k = \alpha_j T_{3,ij}^k - \beta_i + \beta_j + \tau_{i,j} - \alpha_i n_4^k + \alpha_j n_5^k + n_6^k \tag{8}$$

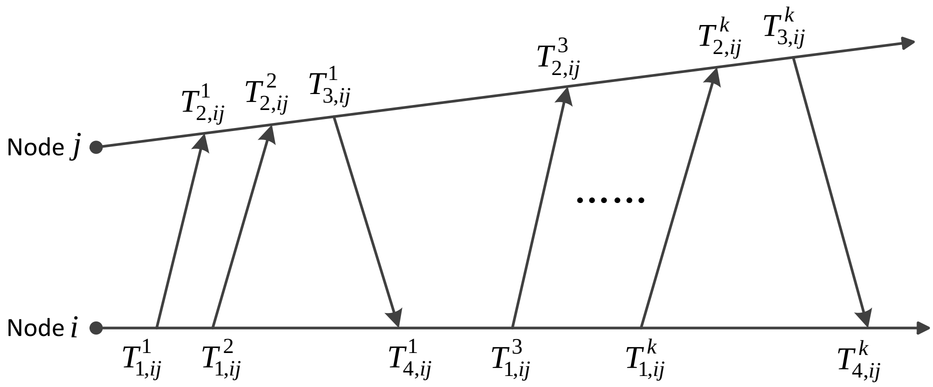

Figure 1. Two way timing message exchange between node *i* and node *j*.

The more links that are collected, the better the estimation performance. Note that there are $X = 2 \times N - 2 + L/2$ unknown variables in total, so at least X communication links must exist if the least square (LS) based solution is to be adopted. In addition, in order to ensure that the matrix is full column rank, at least one single forward link and reverse link with another node must be guaranteed, with one such scenario shown in Figure 2 (b).

Let $L = 2 \bullet \binom{N}{2} = \frac{N!}{(N-2)!}$ be the maximum number of direct communication paths of the network. For the sake of exposition, we assume that each node is capable of communicating with all the other network nodes, and let node 1 be the reference node, namely $\alpha_1 = 1, \beta_1 = 0$, We can extend (7) and (8) to a network wide functional model as

$$\begin{bmatrix} \overbrace{\begin{matrix} -\mathbf{t}_{2,1} & -\mathbf{1}_{2K} & & & & & \\ & & -\mathbf{t}_{3,1} & -\mathbf{1}_{2K} & & & \\ & & & & \ddots & & \\ & & & & \mathbf{t}_{N-1,N} \quad \mathbf{1}_{2K} & -\mathbf{t}_{N,N-1} & -\mathbf{1}_{2K} \end{matrix}}^{2N-2} & \overbrace{\begin{matrix} \mathbf{v}_{2K} & & & \\ & \mathbf{v}_{2K} & & \\ & & \ddots & \\ & & & \mathbf{v}_{2K} \end{matrix}}^{L/2} \end{bmatrix}_{KL\times X} \begin{bmatrix} \mathbf{c}_2 \\ \mathbf{c}_3 \\ \vdots \\ \mathbf{c}_N \\ \boldsymbol{\tau} \end{bmatrix}_{X\times 1} = \begin{bmatrix} \mathbf{t}_{1,2} \\ \mathbf{t}_{1,3} \\ \vdots \\ \mathbf{t}_{1,N} \\ \mathbf{0}_{KL-N+1} \end{bmatrix} + \mathbf{q} \qquad (9)$$

where $\mathbf{t}_{i,j} = [T^1_{1,ij}, T^1_{4,ij}, T^2_{1,ij}, \dots T^K_{4,ij}] \in \mathbb{R}^{2K\times 1}$, $\mathbf{t}_{j,i} = [T^1_{2,ij}, T^1_{3,ij}, T^2_{2,ij}, \dots T^K_{3,ij}] \in \mathbb{R}^{2K\times 1}$, $\mathbf{c}_i = [\alpha_i, \beta_i]^T$ and $\boldsymbol{\tau} = [\tau_{1,2}, \tau_{1,3}, \dots \tau_{N-1,N}]^T$, $\mathbf{q}$ is the vector of noise. Rearranging the terms in (9) we have

$$\mathbf{H}\boldsymbol{\theta} = \mathbf{b} + \mathbf{q} \qquad (10)$$

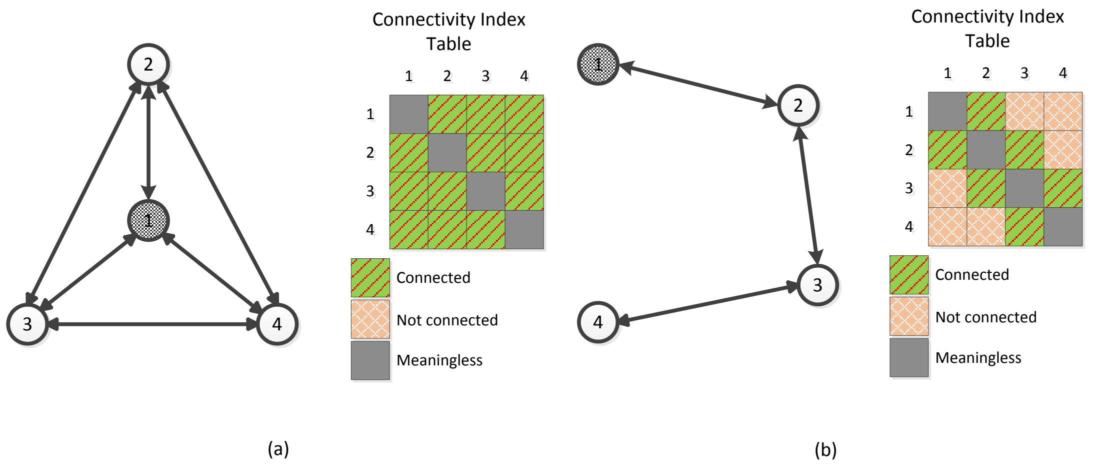

Figure 2. Two feasible examples of the proposed joint estimator for $N = 4$ nodes and related connectivity index tables. (a): Each node is capable of transmitting and receiving timing stamps, thus the best estimation performance could be achieved. (b): For each node, there is only one path to connect with another neighbour node, this is a scenario with the minimum requirement for GLB estimator.

3. PROPOSED ESTIMATOR

In order to solve (9), a GLB estimator (based on sequential least squares (Kay, 2013)) is proposed. This estimator calculates the unknown factors from the perspective view of data fusion, and also in an iterative way. It is expressed by Algorithm 1.

Remark 1. (*Real-time process*): Unlike the existing methods, the estimation is updated once a communication link has been accomplished, namely, $T^k_{2,ij}$ or $T^k_{4,ij}$ is collected. There are two obvious advantage of this design. One is that the computation on the nodes has been dispersed on more time instances. The other advantage is that it is no longer necessary to store the historical information of the network. These two properties become paramount when a restriction of low power consumption exists, such as when sleep wake-up scheduling is implemented.

Remark 2. (*Robustness*): The GLB estimator still estimates the clock skew even if there is only a one-way communication link for an arbitrary child node. This feature makes it more robust than the other existing methods: the estimator will not lose its entire function when the transmitting device or receiving device suddenly stops working. This is because the two-way communication mechanism is capable of distinguishing the clock offset and relative distance, which can be seen from (7) and (8). For instance, if (7) could not be observed, that is to say, only a series of (8) could be used to estimate the clock offset. The term $-\beta_i + \beta_j + \tau_{ij}$ would be considered as an integrated constant for the estimator due to the lack of information. However, the relative clock skew between node i and node j still has a unique solution.

Global Linear Bayesian Estimator

Algorithm 1. GLB estimator

1. **Batch initialization:**
2. Assuming the noise term is uncorrelated and has equal variance $\sigma^2 = \sigma_n^2$ for each data sample, the covariance matrix of the noise $\mathbf{C}[1] \in \mathbb{R}^{X \times X}$ is known and diagonal
3. Collect X communication links in chronological order, initialize with $\mathbf{H}[1] \in \mathbb{R}^{X \times X}$ and $\mathbf{b}[1] \in \mathbb{R}^{X \times 1}$, construct the covariance matrix $\mathbf{\Sigma}[1] = (\mathbf{H}^{\mathrm{T}}[1]\mathbf{C}^{-1}[1]\mathbf{H}[1])^{-1}$
4. Compute the first estimation: $\widehat{\boldsymbol{\theta}}(1) = (\mathbf{H}^{\mathrm{T}}[1]\mathbf{C}^{-1}[1]\mathbf{H}[1])^{-1}\mathbf{H}^{\mathrm{T}}[1]\mathbf{C}^{-1}[1]\mathbf{b}[1]$
5. $m = 1$
6. **Repeat**
7. $m = m + 1$
8. If a new pair of $\{T_{1,ij}^k, T_{2,ij}^k\}$ or $\{T_{3,ij}^k, T_{4,ij}^k\}$ is collected, record $\mathbf{h}[\mathrm{m}] \in \mathbb{R}^{12 \times 1}$ and $\mathbf{b}[\mathrm{m}] \in \mathbb{R}^{1 \times 1}$, which are the new coming information.
9. Calculate the gain matrix $\mathbf{K} \in \mathbb{R}^{X \times 1}$, $\mathbf{K}[\mathrm{m}] = \frac{\mathbf{\Sigma}[\mathrm{m}-1]\mathbf{h}[\mathrm{m}]}{\sigma_n^2 + \mathbf{h}^T[\mathrm{m}]\mathbf{\Sigma}[\mathrm{m}-1]\mathbf{h}[\mathrm{m}]}$.
10. Prediction: $\widehat{\boldsymbol{\theta}}(\mathrm{m}) = \widehat{\boldsymbol{\theta}}(\mathrm{m}-1) + \mathbf{K}[\mathrm{m}]\left(\mathbf{b}[\mathrm{m}] - \mathbf{h}^{\mathrm{T}}[\mathrm{m}]\widehat{\boldsymbol{\theta}}(\mathrm{m}-1)\right)$.
11. Update covariance: $\mathbf{\Sigma}[\mathrm{m}] = (\mathbf{I} - \mathbf{K}[\mathrm{m}]\mathbf{h}^{\mathrm{T}}[\mathrm{m}])\mathbf{\Sigma}[\mathrm{m}-1]$.
12. **Until** the latest record has been processed

Remark 3. (*Computation efficiency*): In practice, it would take a number of memory footprints to calculate the inverse of a matrix. Furthermore, the computation complexity is proportional to the size of matrix. As discussed in Section 2.1, the minimum size of information matrix is X rows and X columns, whereas normally several times the columns with respect to X are required regarding the estimation precision. It can be found that the GLB estimator can reduce the observation information matrix to the size of X rows and 1 column.

4. Numerical simulations

A scenario in Figure 2 (a) is provided to evaluate the performance of the GLB estimator compared with the other existing solutions. All the nodes are located in a 2-dimensional space, whose locations are uniform distributed variables in the region of $10km \times 10km$, expressed as $\mathbf{P} = \{\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3, \mathbf{P}_4\} \in \mathbb{R}^{2 \times 4}$. The propagation delay τ_{ij} is then $\tau_{\mathrm{ij}} = v^{-1} \cdot \left\|\mathbf{P}_i - \mathbf{P}_j\right\|_2$ seconds, where $v = 1km/s$ is the speed of data rate. The clock skews and clock offsets of the child nodes are uniform randomly distributed in the range $[-1, 1]$ and $[1 - 2ppm, 1 + 2ppm]$ seconds respectively. The independent transmitting times $T_{2,ij}^k$ and $T_{4,ij}^k$ are sorted uniform randomly distributed variables in the range $[1, 500]$ seconds. The noise deviations of time stamps and propagation delays are 0.1 milliseconds and 0.5 milliseconds respectively. A performance matrix named average root mean square error (ARMSE) is employed to evaluate the performance of the proposed estimator (Schenato et al., 2014). All the given results are averaged over 1000 Monte Carlo runs.

Figure 3, Figure 4 and Figure 5 show the ARMSEs of the unknown clock skews, clock offsets and relative distances respectively. The reason why we choose GLS and LCLS as the comparisons is that the two estimators have better performances than the other estimators. It can be seen that the estimation performances of GLB and GLS for the three parameters are nearly the same, and both of them outperforms the LCLS.

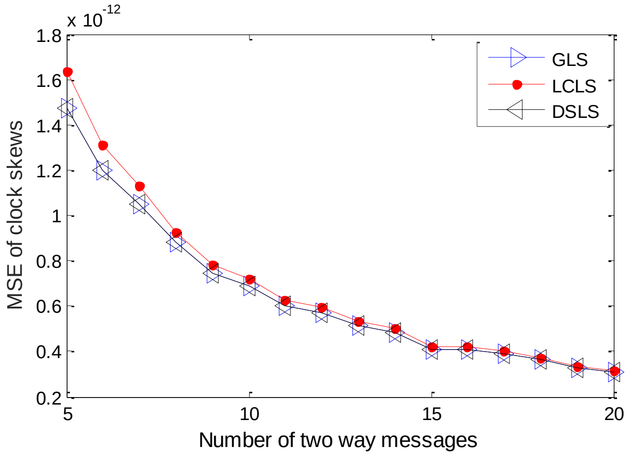

Figure 3. ARMSE of the estimated clock skews

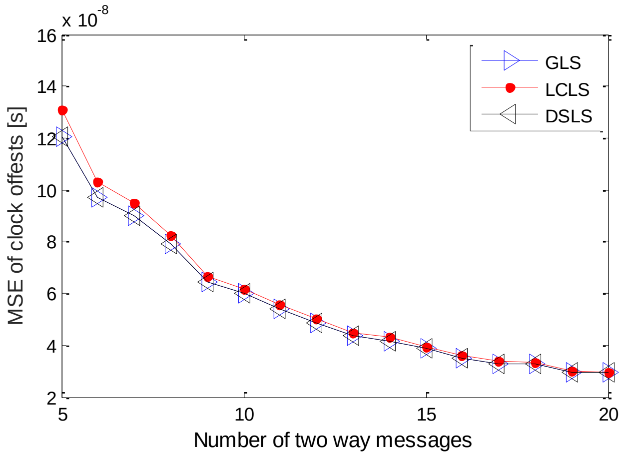

Figure 4. ARMSE of the estimated clock offsets

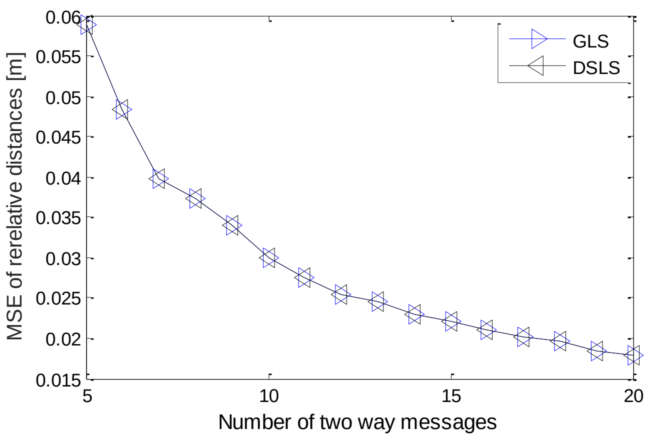

Figure 5. ARMSE of the estimated relative distances

5. CONCLUSIONS

The methods in (Leng & Wu, 2010) and (Noh et al., 2007) consider the vector $\boldsymbol{\tau}$ as a nuisance term and simplify the matrix by differencing or adding the observations. To realize this, a symmetrical timing stamps exchange mechanism has to be put forward. However, the global least square (GLS), proposed in (Rajan & Van der Veen, 2011) overcomes this restriction and outperforms the other existing algorithms according to the simulation results. It considers (9) as a linear LS estimation. However, the GLS solution brings the problem of computation complexity especially when N is large. The GLB estimator proposed in this paper effectively solves these problems. The updating time has been greatly reduced, as well as the computational complexity. Meanwhile, its estimation precision is as good as the best performance of existing estimators. Future research directions include the extension of realizing recursive clock synchronization and localization.

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