A geometry-free approach to reliable GNSS triple carrier ambiguity resolution over hundreds of kilometers

Yanming Feng, Charles Wang
Queensland University of Technology, Australia
Qile Zhao, Wuhan University, China

IGNSS 2015, July 16, 2015

Contents

1. The challenges in network-based multi-GNSS data processing
2. Geometry-free models for DD integer ambiguity resolution
3. Network-based geometry-free AR and reliability control
4. Experiential results and AR performance analysis
5. Summary

1. Challenges of network-based GNSS data processing

- **A key to network-based GNSS data processing is to recover the integer ambiguities of all DD phase measurements**
- **The current approach to DD ambiguity resolution (AR)**
  1. Use the geometry-free /ionosphere-free (GFIF) combination to fully fix the wide-lane (WL) integers
  2. Use the geometry-based models to estimate the narrow-lane (NL) float ambiguities and derive the covariance matrix in the adjustment
  3. Use integer rounding (IR) or integer bootstrapping (IB) to *fully or partially* resolve the NL integers
- **The challenges:**
  1. About 2/3 of the parameters being estimated are ambiguity parameters (eg. GAMIT)
  2. AR capability could be distance-dependent or unreliable
  3. For a large scale network of more than one hundred stations, the estimation is computationally very intensive

1. The Key to network-based GNSS data processing

- The proposed geometry-free approach for triple frequency AR includes the following procedures
  1. Geometry-free/ionosphere-free models (GFIF)
     - Two Wide-lanes (WL)
     - One Narrow-lane (NL)
  2. Resolve all WL and NL DD integers over any distances
  3. Map the DD integers into the UD integers by appending the necessary rows to the transformation matrix

2 GFIF models for DD integer estimation

Virtual code and phase signals

$$P_{12} = \frac{f_1 P_1 + f_2 P_2}{f_1 + f_2}, \phi_{12} = \frac{f_1 \phi_1 - f_2 \phi_2}{f_1 - f_2}, \quad (1)$$

$$P_{25} = \frac{f_2 P_2 + f_5 P_5}{f_2 + f_5}, \phi_{25} = \frac{f_2 \phi_2 - f_5 \phi_5}{f_2 - f_5} \quad (2)$$

Two wide-lane GFIF combinations (after double-difference)

$$\Delta\nabla P_{12} - \Delta\nabla \phi_{12} = \lambda_{12} \Delta\nabla N_{12} + \varepsilon_{\Delta\nabla P_{12}} - \varepsilon_{\Delta\nabla \phi_{12}} \quad (3)$$

$$\Delta\nabla P_{25} - \Delta\nabla \phi_{25} = \lambda_{25} \Delta\nabla N_{25} + \varepsilon_{\Delta\nabla P25} - \varepsilon_{\Delta\nabla \phi_{25}} \quad (4)$$

The carrier-phase only Narrow-lane combinations

$$\begin{aligned} &\Delta\nabla \phi_1 - (\alpha_{11} \Delta\nabla \phi_{12} + \alpha_{12} \Delta\nabla \phi_{25}) - \alpha_{11} \lambda_{12} \Delta\nabla N_{12} - \alpha_{12} \lambda_{25} \Delta\nabla N_{25} \\ &= -\lambda_1 \Delta\nabla N_1 + \varepsilon_{\Delta\nabla \phi_1} - \alpha_{11} \varepsilon_{\Delta\nabla \phi_{12}} - \alpha_{12} \varepsilon_{\Delta\nabla \phi_{25}} \end{aligned} \quad (5)$$

2 DD GFIF models for integer estimation

DD GFIF observable	Ambiguity	GPS signals		BDS signals	
		σ^2	σ $\sigma_P = 0.3m$ $\sigma_\phi = 2mm$	σ^2	σ $\sigma_P = 0.3m$ $\sigma_\phi = 2mm$
$\Delta\nabla P_{12} - \Delta\nabla\phi_{12}$	$\Delta\nabla N_{12}$	$2.03\sigma_P^2 + 132\sigma_\phi^2$ $\lambda_{(1,-1,0)} = 0.862m$	0.425 m **(0.609 cycles)**	$2.02\sigma_P^2 + 189\sigma_\phi^2$ $\lambda_{(1,-1,0)} = 1.0247m$	0.425 m **(0.415 cycles)**
$\Delta\nabla P_{25} - \Delta\nabla\phi_{25}$	$\Delta\nabla N_{25}$	$2\sigma_P^2 + 4420\sigma_\phi^2$ $\lambda_{(0,1,-1)} = 5.82m$	0.468 m **(0.080 cycles)**	$2\sigma_P^2 + 3256\sigma_\phi^2$ $\lambda_{(0,1,-1)} = 4.8842m$	0.4394m **(0.090 cycles)**
$\Delta\nabla\phi_1 - \alpha_{11}\Delta\nabla\phi_{12}$ $-\alpha_{12}\Delta\nabla\phi_{25} + \lambda_{25}\alpha_{11}\Delta\nabla N_{25}$ $+\lambda_{25}\alpha_{12}\Delta\nabla N_{25}$	$-\Delta\nabla N_1$	$385^2\sigma_\phi^2$ $\lambda_{(1,0,0)} = 0.1903m$	0.752 m **(4.05 cycles)**	$407^2\sigma_\phi^2$ $\lambda_{(1,0,0)} = 0.1920m$	0.8141 m **(4.24 cycles)**
Remarks:		$\alpha_{11} = \frac{f_1^2 + f_2 f_5}{f_1^2 - f_1 f_5} = 6.2464,$ $\alpha_{12} = 1 - a_{11} = -5.2464$		$\alpha_{11} = \frac{f_1^2 + f_2 f_5}{f_1^2 - f_1 f_5} = 7.1816$ $\alpha_{12} = 1 - a_{11} = -6.1816$	

To achieve 99% success rate, the STD should be 0.2 cycles. It takes about 400 independent samples to converge to this accuracy.

3. Network-based geometry-free AR and reliability

- The GFIF phase-only narrow-lane method:
  - Float solutions are obtained by averaging samples over a few hours
  - Covariance matrices can be computed over independent baselines
    - Allowing the Integer Bootstrapping (IB) to be used
    - Allowing all the existing integer acceptance testing procedures to be applied
- Geometry-free (GF) observation vector for all DD pairs:

$$\Delta\nabla\mathbf{GF}(t) = \lambda\Delta\nabla\mathbf{N} + \Delta\nabla\boldsymbol{\varepsilon}(t)$$

- Float solution:

$$\Delta\nabla\tilde{\mathbf{N}} = \frac{1}{\lambda T}\sum_{t=1}^{T}\Delta\nabla\mathbf{GF}(t)$$

- Covariance matrix: computed from the residual time series

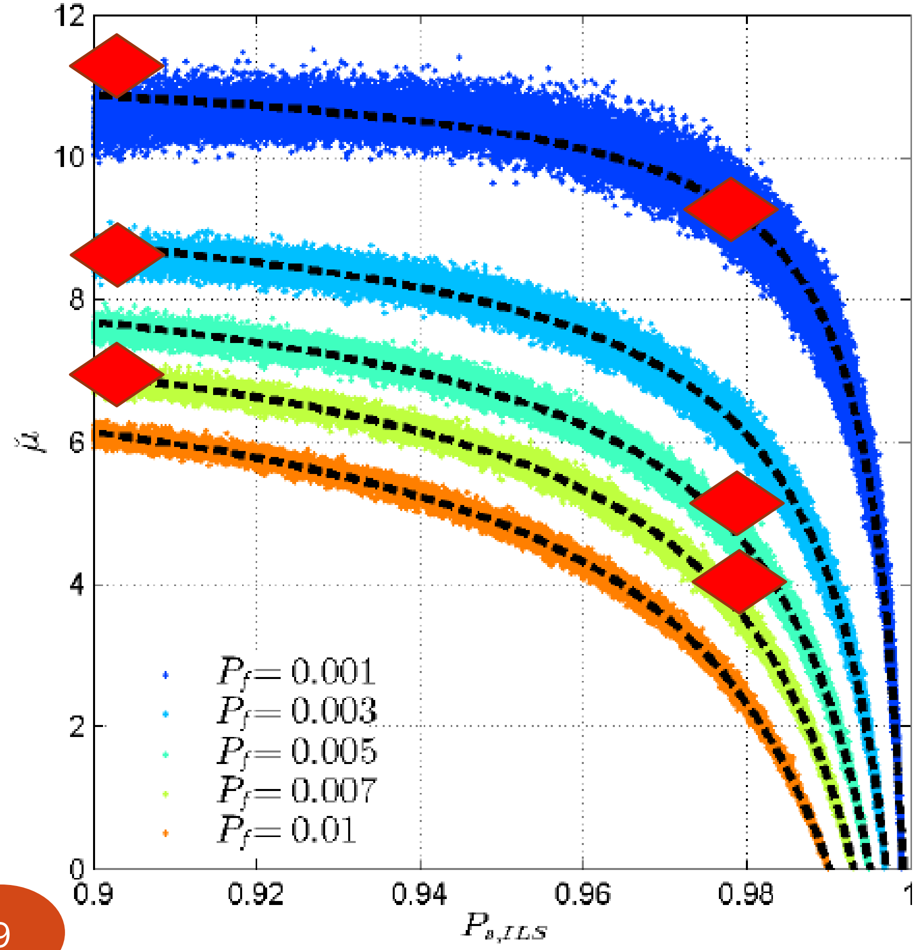
$$Cov(\Delta\nabla\tilde{\mathbf{N}}) = \frac{1}{\lambda_I^2 T(T-1)}\sum_{t=1}^{T}[\Delta\nabla\mathbf{GF}(t) - \lambda_I\Delta\nabla\tilde{\mathbf{N}}]\left[\Delta\nabla\mathbf{GF}(t) - \lambda_I\Delta\nabla\tilde{\mathbf{N}}\right]^T$$

3.1 Ambiguity validation: step 1 stability/stationarity

- Stability/stationarity verification of ambiguity solutions
  - IR, IB and ILS estimators give the same results when the success rate is very high
  - The float solutions remain stable after convergence or the integer solution remain unchanged beyond a sufficiently large sample size
  - The integer solutions agree with the solutions from other methods
    - Depending on how the integers are defined and determined

3.2 Ambiguity validation step 2: Acceptance tests

Integer Least square (ILS) solution passes the Difference test at an given success rate:



$$\left\|\hat{\mathbf{a}}-\breve{\mathbf{a}}_2\right\|^2_{\mathbf{Q}_{\hat{a}\hat{a}}} - \left\|\hat{\mathbf{a}}-\breve{\mathbf{a}}_1\right\|^2_{\mathbf{Q}_{\hat{a}\hat{a}}} \geq \mu$$

where

$\hat{\mathbf{a}}$ is the float vector;

$\breve{\mathbf{a}}$ is a integer vector;

$\mathbf{Q}_{\hat{a}\hat{a}}$ is the covariance matrix

- This is to ensure the success rate or the failure rate
- If the computed success rate is high, the testing thresholds can be very low.
- If the covariance matrix is realistic, the computed success rate is close to the actual success rate
- If tests are undecided, use the float solutions as the initial solutions in the follow-on geometry-based estimation

Wang Lei (2015), "Reliability control of GNSS carrier-phase integer ambiguity resolution", QUT PhD thesis

4. Numerical results and AR performance analysis

4.1 The experimental site map and data sets

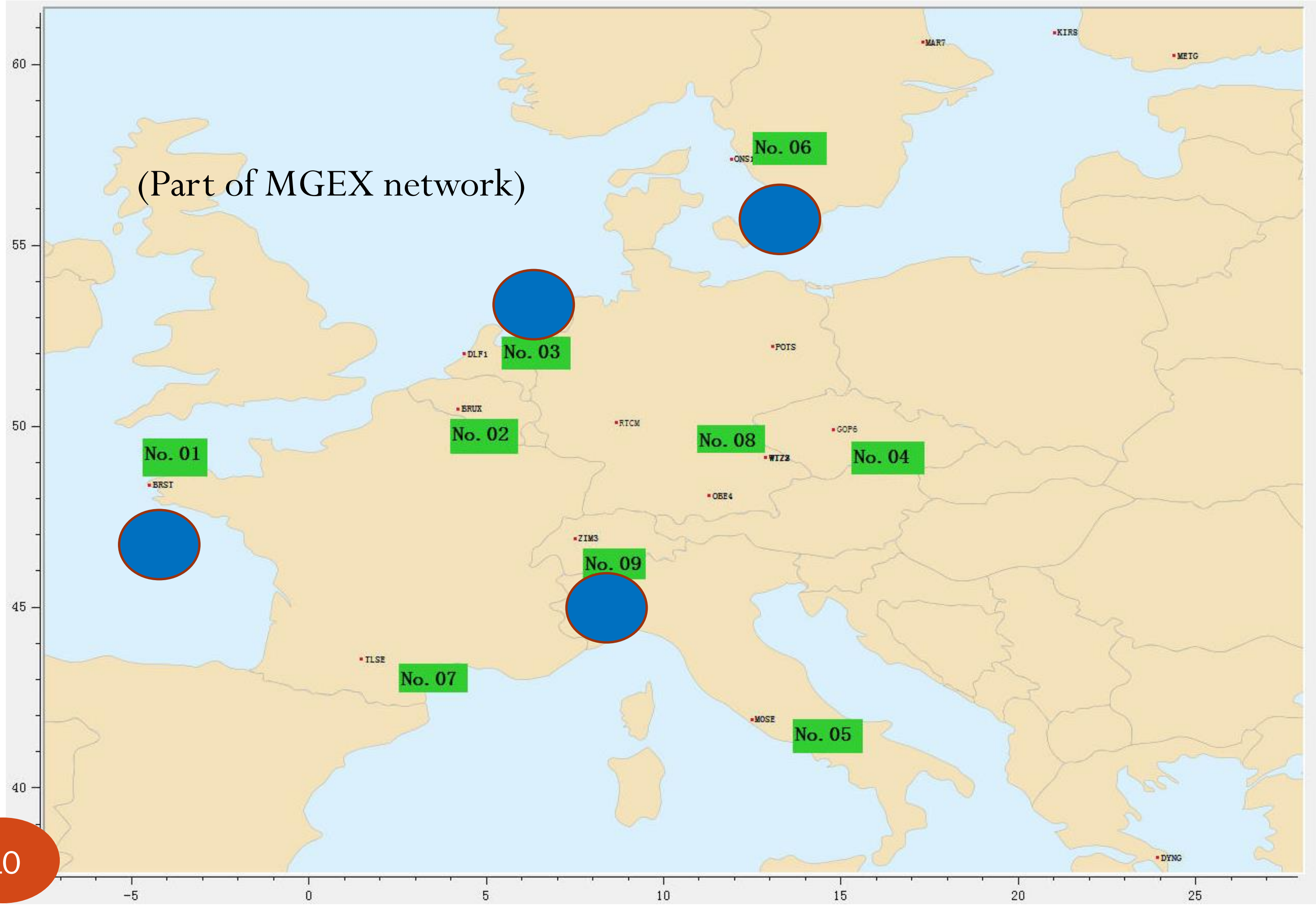


Table 2. Information of data sets for experimental analysis (2014)

No	Station ID	Day 299	Day 300	Day 301
1	BRST	Interval 01:28:44 ~ 05:11:14 3h42min Sample rate: 30 seconds	Interval 01:46:44 ~ 05:06:14 3h20min Sample rate: 30 seconds	Interval 01:20:44 ~ 05:03:14 3h43min Sample rate: 30 seconds
2	BRUX			
3	DLF1			
4	GOP7			
5	MOSE			
6	ONS1			
7	TLSE			
8	WTZ8			
9	ZIM3			

Stations that tracked Beidou satellites

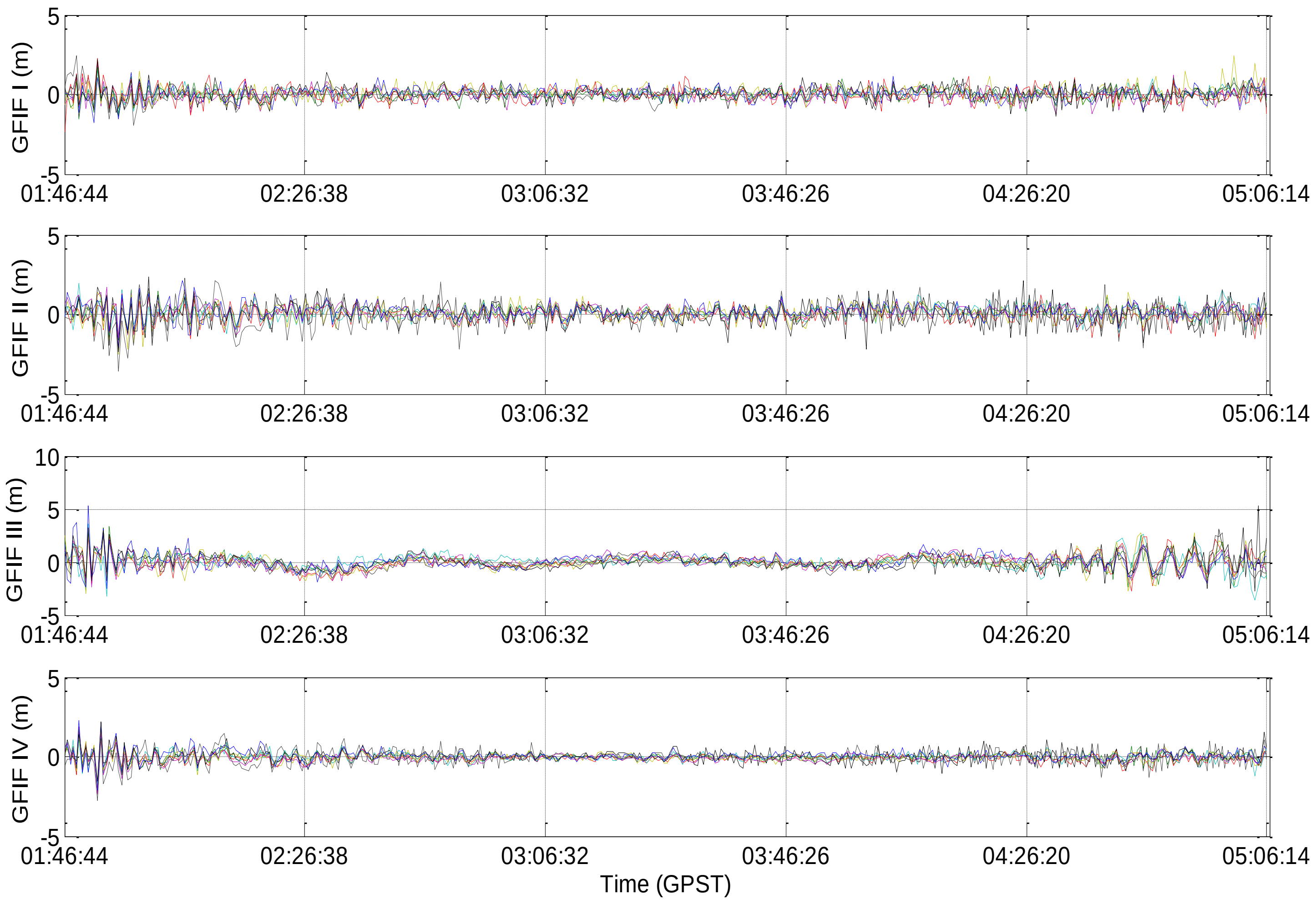
4.2 GFIF noise characteristics and analysis (GPS)

Table 3 Standard deviation of UD and DD wide-lane and narrow-lane GFIF observables in metre and cycle units

Satellite pair	STD Type	GFIF I	λ=0.862m	GFIF II	λ=5.861m	GFIF III	λ=.190m	GFIF IV
		metre	cycle	metre	cycle	metre	Cycle	metre
PRN24	DD STD	0.4470	0.5186	0.5514	0.0941	0.6823	3.5853	0.3361
PRN25	UD STD	0.2128	0.2469	0.2685	0.0458	0.3390	1.7813	0.1609
PRN06	DD STD	0.4154	0.4819	0.5123	0.0874	0.6252	3.2852	0.2959
PRN09	UD STD	0.1948	0.2260	0.2346	0.0400	0.2899	1.5236	0.1353

Statistical results of DD GFIF observables over 36 baselines and 18 UD GFIF observables are given. The STD values of GFIF UD and DD errors agree to each other well in terms of the variance propagation

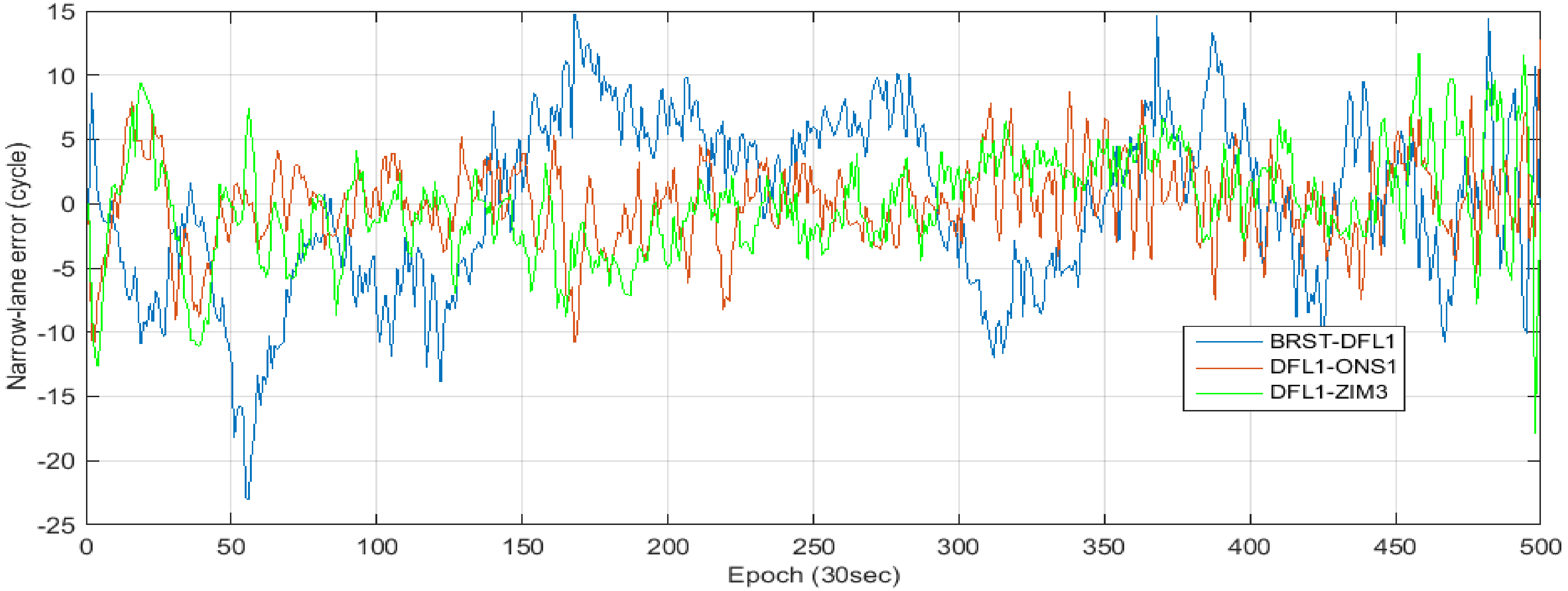
4.3 GFIG DD residuals after removal of integers



The figure plots the errors of DD GFIF models I to IV in four panels for eight selected independent baselines with the satellite PRN24 and PRN25, showing the generally random characteristics similar to the well-understood GFIF model I over a long data period, but the NL suffers more from the amplified phase multipath effects.

Beidou DD noise levels and characteristics

		BRST-DFL1	BRST-ONS1	BRST-ZIM3	DFL1-ONS1	DFL1-ZIM3	ONS1-ZIM3
Day299	C11-C12	5.5100	5.5215	5.1879	2.8667	3.1524	3.0107
Day301	C6-C11	7.0728	6.9830	7.0647	3.3331	3.7501	3.5162

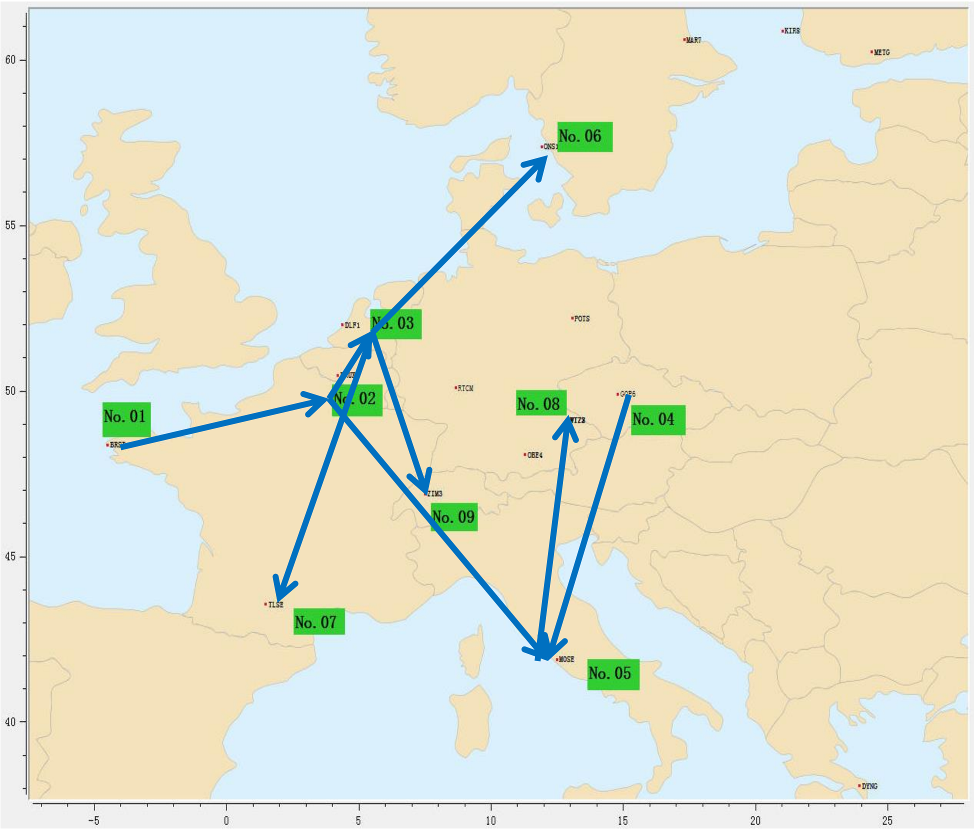


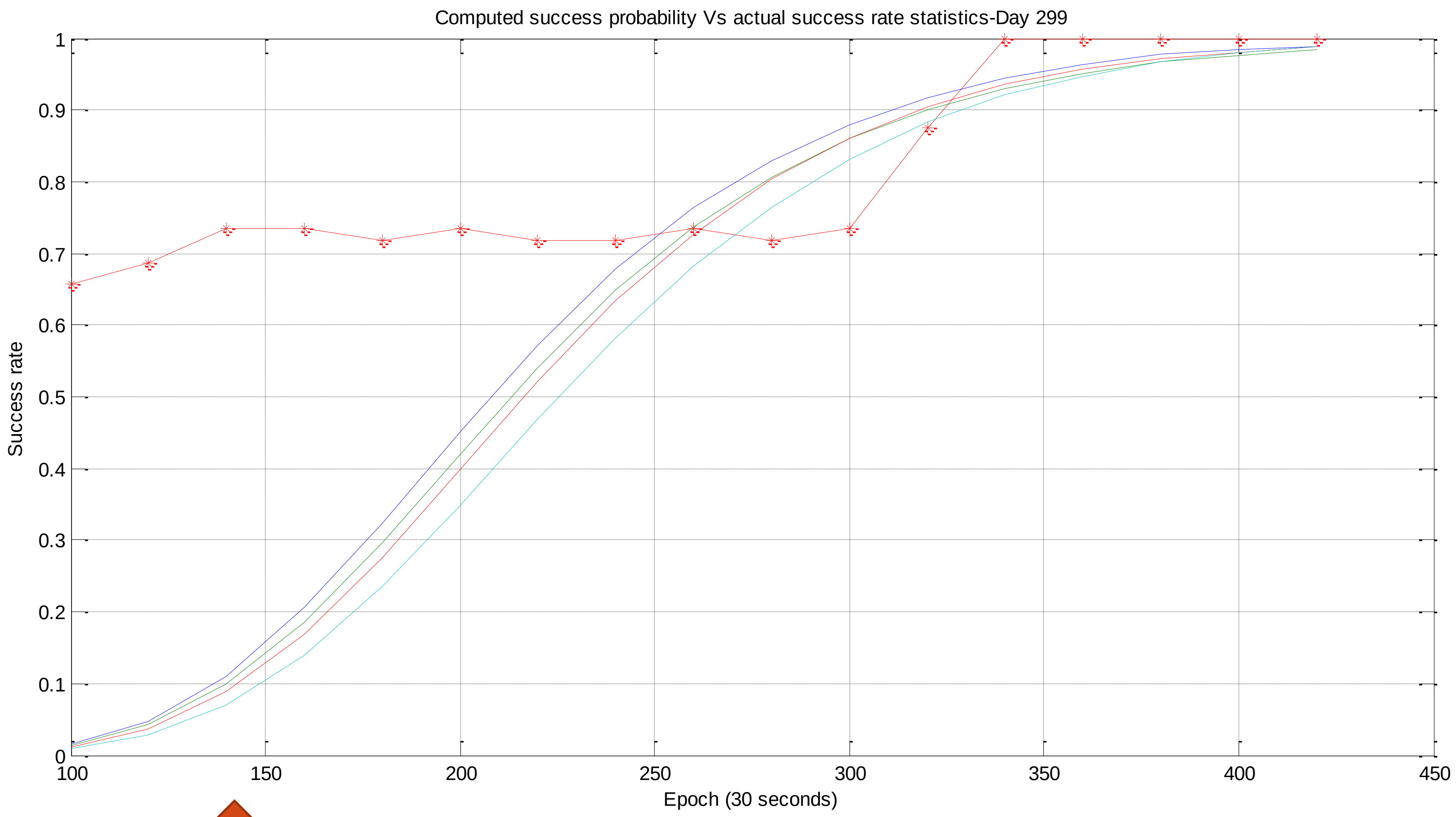
4.4 NL integer rounding results over 8 selected baselines

Float ambiguity solutions of 8 independent baselines with PRN24 and PRN25 and integer rounding success rate

From	To	Distance (km)	NL bias to nearest integer (0.190m)	NL STD (0.190m)	NL Success probability
BRST	BRUX	692	**-0.0290**	**0.1878**	0.9915
BRUX	DLF1	132	**0.1066**	**0.1390**	0.9977
BRUX	MOSE	1169	**-0.0696**	**0.1310**	0.9995
DLF1	ONS1	771	**0.1919**	**0.1470**	0.982
DLF1	TLSE	961	**0.1080**	**0.1326**	0.9984
DLF1	ZIM3	610	**0.0250**	**0.1910**	0.9906
GOP7	MOSE	909	**0.0608**	**0.1847**	0.9901
MOSE	WTZ8	807	**-0.0898**	**0.1591**	0.9943

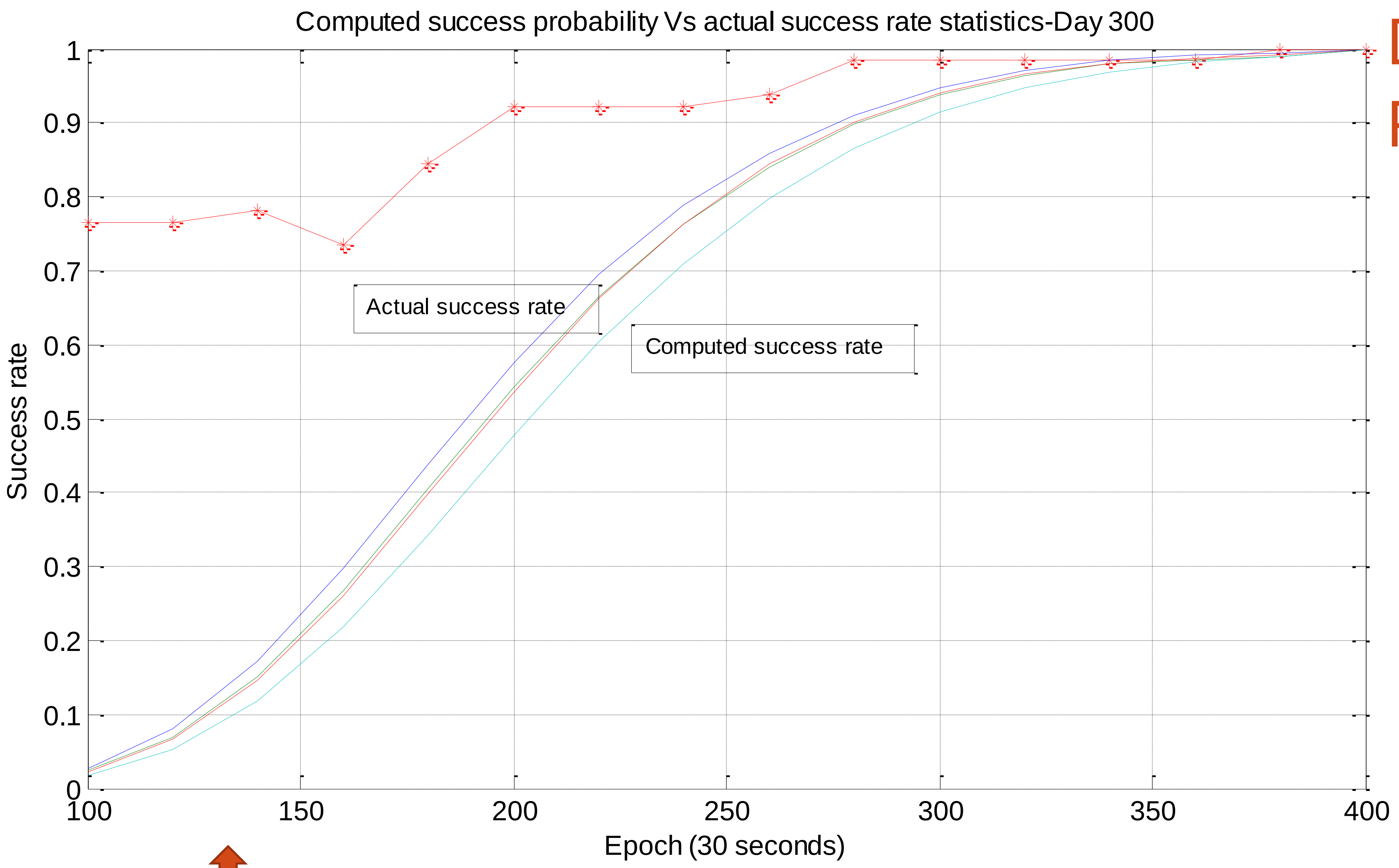
8 independent baselines (with highest success rate)





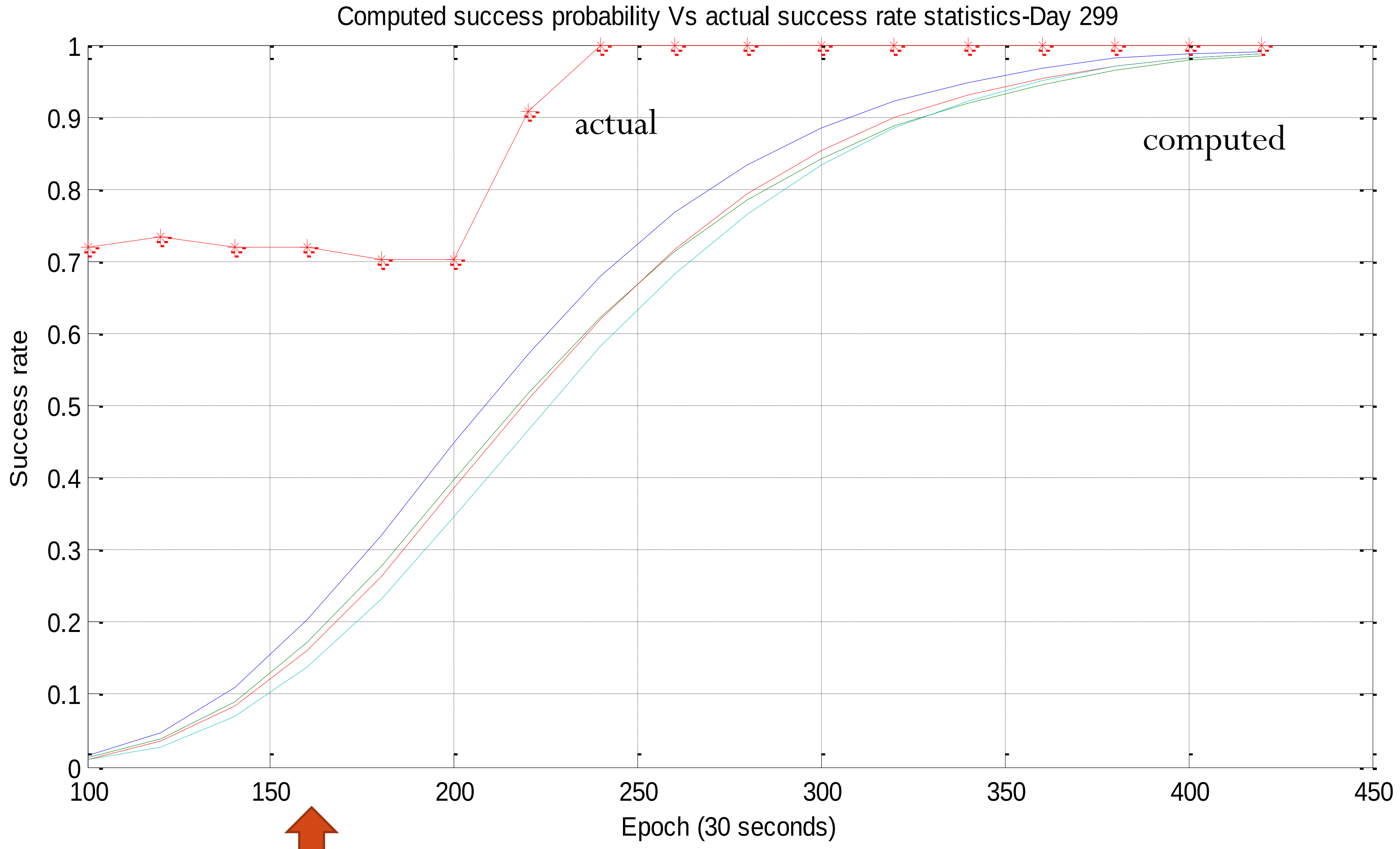
Day 299
PRN24-PRN25

Comparison of computed IB success rate and actual success rate over 4x8 baselines



Day 300 PRN24-PRN25

Comparison of computed IB success rate and actual success rate over 4x8 baselines



Day 301 PRN24-PRN25

Comparison of computed IB success rates and actual success rate over 4x8 baselines

4.5 Remarks on DD integer AR reliability

- The success rate of the NL integer solutions is distance-independent
- The actual success is often higher than the computed IB success rate
- When the computed success rate is higher than 95%, the computed success rate is sharply close to the actual success rate
- The integer solution remain unchanged after certain epochs (eg. 250 min the Day 301 data set).
- In general, multiple strategies may be adopted to verify the AR reliability from different accepts.

5. Summary

- **With the phase-only GFIF combination, the NL DD noise level is about 3-4 cycles, allowing the integers to be reliably fixed over the certain long data arcs**

- **The reliability of GFIF integer ambiguity resolution may be controlled through various conditions**

- **The numerical analysis shows that the DD NL integer ambiguities of eight baselines of up to 1170 km in length are fixed to their integers by IR and IB at the success rates of 98-99.9%. The actual success rate is 100% for all NLs with the tested data sets of 400 epochs.**

- **Overall, with the geometry-free model, the NL DD integer AR capability is reliable and distance-independent. This can significantly reduce the network-based GNSS data processing complexity**

Thank you!